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13. ABSTRACT (Maximum 200 words)

Here we submit the final report for research carried out at the Department of Statistics, University of Missouri-Columbia during the period October 1, 1992 - July 31, 1994. During this period the Principal Investigator (PI), A. P. Basu, along with his collaborators considered a number of problems in reliability theory. A. P. Basu edited the research monograph "Advances in Reliability" (1993), which has been published by North-Holland. Six refereed papers also were published. The titles of these papers are given below.

1. Life Testing and Reliability Estimation with Asymmetric Loss Function
2. On a test for exponentiality against Monotone Failure Rates
3. Bayesian Approach to Some Problems in Life Testing and Reliability Estimation
4. Characterizations of a Family of Bivariate Exponential Distributions
5. Bayesian Reliability of Stress-Strength Systems
6. Some Problems of "Safe Dose" Estimation

Some of the research areas are of interest to researchers at Rome Laboratory, Griffiss Air Force Base, New York.

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Summary of Primary Research Activities

The Principal Investigator edited the research monograph "Advances in Reliability" and published six refereed papers. The major problems considered deal with reliability estimation of complex systems. Also, the stress-strength model is considered, where a component of random strength x is subjected to a random stress y. Bayesian estimation is considered as the Bayesian paradigm allows us to incorporate prior knowledge and expert opinion. In addition to squared error loss, asymmetric loss function is considered. Asymmetric loss function is especially appropriate when overestimation of reliability is much more serious than underestimation. For example, overestimation of solid-fuel rocket booster reliability led to the 1986 disaster of the US space shuttle Challenger. Properties of the estimates are obtained. The suitability of using the univariate and multivariate exponential distributions as models has also been considered. The total failure rate concept is introduced, and a number of characterization theorems have been proven. Tests for exponentiality are also considered against the alternative that the underlying models have monotone failure rates.

Efficient Composite Designs with Small Number of Runs

by

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0 Summary

A new method of constructing composite designs using the robustness property against deletion of runs (points) is given. Composite designs are presented with small number of runs. These designs are more efficient in terms of both prediction of response and estimation of parameters, than their competitors that are available in the literature.

Short Running Title: Composite Designs

Key Words: Deletion of Points, Factorial Points, Orthogonal, Response Surface, Robustness, Unbiasedness.

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1 Introduction

In the study of dependence of a response variable y on k explanatory variables, coded as $x_1, \ldots, x_k$, the unknown response surface is approximated by a first or a second order polynomial in a small region with the center being the point of maximum interest. If the first order model gives a significant lack of fit indicating the presence of a surface curvature, a second order response surface model is then fitted. The N runs (or points) in the design are $(x_{u1}, \ldots, x_{uk})$ and the observations are $y(x_{u1}, \ldots, x_{uk}) = y_u,\ u = 1, \ldots, N$. The expectation

of y_u under the second order model is

$$E(y_u) = \beta_o + \sum_{i=1}^{k} \beta_i x_{ui} + \sum_{i=1}^{k} \beta_{ii} x_{ui}^2 + \sum_{\substack{i=1 \\ i<j}}^{k} \sum_{j=1}^{k} \beta_{ij} x_{ui} x_{uj}, \tag{1}$$

where the intercept β_o, the linear coefficients β_i, the pure quadratic coefficients β_{ii} and the interaction coefficients β_{ij} are unknown constants. The y_u's are assumed to be uncorrelated with the variance σ^2, an unknown constant. The number of β's in (1) is $1 + 2k + \binom{k}{2}$. For a second order design with N points $(x_{u1}, \ldots, x_{uk}), u = 1, \ldots, N$, all β's are unbiasedly estimable.

A special second order design, called composite design (CD), consists of F factorial points (FP's) which are a fraction of 2^k points $(\pm 1, \ldots, \pm 1)$, $2k$ axial points (AP's) $(\pm\alpha, \ldots, 0), \ldots, (0, \ldots, \pm\alpha)$, α is a given constant and $n_o (\geq 0)$ center points (CP's) $(0, \ldots, 0)$. The total number of points is $N = F + 2k + n_o$. Box and Wilson (1951) introduced such designs. Box and Hunter (1957) suggested FP's as the complete set of 2^k points or an orthogonal resolution V plan (i.e, the plan that permits the unbiased estimation of $\beta_o + \sum_{i=1}^{k} \beta_{ii}, \beta_i$'s and β_{ij}'s under (1) and, moreover, the estimators are uncorrelated). The CD's with such FP's give the variance of the predicted response dependent on the point only through its distance from the origin. This variance structure is achieved at the cost of a large number of points, particularly FP's. Efforts are then being made for reducing the number of FP's. Hartley (1959) pointed out that FP's need not be of resolution V but could be as low as of resolution III plan (i.e., the plan that permits the unbiased estimation of $\beta_o + \sum_{i=1}^{k} \beta_{ii}$ and β_i's assuming β_{ij}'s are known) with an additional condition that the unbiased estimation of β_{ij}'s is possible assuming the other β's are known. Draper and Lin (1990) named such FP's as resolution III*. Hartley (1959) presented resolution III* FP's as regular fractions and Westlake (1965) presented FP's as irregular fractions of 2^k factorials. Draper (1985), Draper and Lin (1990) gave resolution III* FP's using the projection properties of Plackett and Burman orthogonal resolution III fractions of 2^k factorials.

In this paper, a new method of constructing CD's is given by introducing first a submodel of (1), presenting orthogonal FP's under the submodel and finally, reducing the number of FP's using the idea of robustness of designs against deletion of points [see Ghosh (1979)]. New CD's constructed by this method, can be made minimal in the number of FP's. They are then compared with the available CD's in the literature. Comparisons are made with respect to the Trace, Determinant and Maximum Characteristic Root of the variance-covariance matrix of the least squares estimators of β's and also with respect to the average of the variances of predicted responses in a spherical region about the center. Our designs perform significantly better over the designs that are available in the literature. Results presented in this paper are striking, useful and valuable.

2 Factorial Points

Observations at AP's permit the unbiased estimation (UE) of β_i and $\beta_o + \alpha^2\beta_{ii}$, $i = 1,\ldots,k$. The CP observations provide the UE of β_o. For a second order CD, observations at FP's must at least allow the UE of $\beta_o + \sum_{i=1}^{k}\beta_{ii}$ and β_{ij}, $i < j, i, j = 1,\ldots,k$, given that the estimators of $\beta_1,\ldots,\beta_k$ are available from AP's. When there is no center point observation (i.e., $n_o = 0$), α can not be equal to $k^{1/2}$. In view of this, the following submodel of (1) is introduced for the choice of FP's.

$$E(y_u) = \beta_o + \sum_{\substack{i=1 \\ i<j}}^{k}\sum_{j=1}^{k}\beta_{ij}x_{ui}x_{uj},\ u = 1,\ldots,F. \tag{2}$$

In matrix notation,

$$E(\underline{y}) = X\underline{\beta},\ V(\underline{y}) = \sigma^2 I, \tag{3}$$

where $\underline{y}(F \times 1)$ is the vector of observations at FP's, $\underline{\beta}(p \times 1), p = 1 + \binom{k}{2}$, is the vector of β's in (2) and $X(F \times p)$ is the design matrix based on FP's. It is important to note that a CD with $\alpha > 0$ for $n_o \geq 0$ and in addition, $\alpha \neq k^{1/2}$ for $n_o = 0$ is of second order if and only

if the UE of all β's under (2) is possible (i.e., Rank $X = p$).

A set of FP's is said to be orthogonal if $X'X$ is a diagonal matrix and is denoted by OFP's. FP's which are not OFP's are called nonorthogonal FP's (NOFP's). Notice that if one FP is negative of another FP, then the corresponding rows of X are identical.

Let F_w be a set of FP's with the corresponding vector of observations $\underline{y}_w$, $w = 1, 2$, and $\underline{\beta}' = (\underline{\beta}_1', \underline{\beta}_2', \underline{\beta}_3')$, where $\underline{\beta}_i(p_i \times 1)$, $i = 1, 2, 3$, with $p_1 + p_2 + p_3 = p$. Denote

$$E(\underline{y}_w) = X_{w1}\underline{\beta}_1 + X_{w2}\underline{\beta}_2 + X_{w3}\underline{\beta}_3, \tag{4}$$

where $X_{wi}(F_w \times p_i), w = 1, 2, i = 1, 2, 3$. Let F_1 and F_2 FP's be such that

$$X_{12} = X_{13}, X_{22} = -X_{23}, \; p_2 = p_3. \tag{5}$$

Example 1. For $k = 8$, the 64 FP's satisfying $x_1x_2x_3 = x_4x_5x_6 = -1$ form OFP's. For $F_1 = 32$ FP's satisfying $x_1x_2x_3 = x_4x_5x_6 = -1$, $x_1x_4x_7x_8 = 1$ and $F_2 = 32$ FP's satisfying $x_1x_2x_3 = x_4x_5x_6 = x_1x_4x_7x_8 = -1$, $\underline{\beta}_2' = (\beta_{14}, \beta_{17}, \beta_{47})$ and $\underline{\beta}_3' = (\beta_{78}, \beta_{48}, \beta_{18})$, $p_2 = p_3 = 3$, $p_1 = 23$, the conditions in (5) in fact hold. The following result is very useful in the selection of FP's.

Theorem 1. If for F_1 FP's Rank $[X_{11}, X_{12}] = p_1 + p_2$ and for F_2 FP's Rank $X_{22} = p_2$ then for $(F_1 + F_2)$ FP's,

$$\text{Rank} \begin{bmatrix} X_{11} & X_{12} & X_{13} \\ \\ X_{21} & X_{22} & X_{23} \end{bmatrix} = p.$$

Proof. There exists a $p_2 \times p_2$ submatrix X_{22}^* of X_{22} with Rank $X_{22}^* = p_2$. The matrix obtained by taking the corresponding p_2 rows of $[X_{21}, X_{22}, X_{23}]$ is $[X_{21}^*, X_{22}^*, X_{23}^*]$. It now remains to prove that

$$\text{Rank} \begin{bmatrix} X_{11} & X_{12} & X_{13} \\ \\ X_{21}^* & X_{22}^* & X_{23}^* \end{bmatrix} = p.$$

Let if possible that rank be $(p-s)$, where $s>0$. There exists an $((F_1+F_2-p+s)\times(F_1+F_2))$ matrix $[D_1, D_2]$ with rank equals to F_1+F_2-p+s and satisfying

$$[D_1, D_2]\begin{bmatrix} X_{11} & X_{12} & X_{13} \\ \\ X^*_{21} & X^*_{22} & X^*_{23} \end{bmatrix} = 0.$$

Thus $D_1X_{12}+D_2X^*_{22}=0$ and, moreover, $D_1X_{13}+D_2X^*_{23}=D_1X_{12}-D_2X^*_{22}=0$ (since, from (5), $X_{12}=X_{13}$ and $X^*_{22}=-X^*_{23}$). This implies that $D_2X^*_{22}=0$ and consequently $D_2=0$. Then Rank $D_1=(F_1+F_2-p+s)>(F_1-p_1-p_2)$ and this is impossible since Rank $[X_{11}, X_{12}, X_{13}]=p_1+p_2$. This completes the proof.

<u>Example 2.</u> For $k=8$ and $F_1=32$ FP's satisfying $x_1x_2x_3=x_4x_5x_6=-1$ and $x_1x_4x_7x_8=1$, Rank $[X_{11}, X_{12}]=p_1+p_2=23+3=26$. From (5), $X_{22}=-X_{23}$ and the columns of X_{23} correspond to β_{18}, β_{48} and β_{78}. It is now clear that the condition Rank $X_{22}=p_2=3$ in Theorem 1 can be achieved by choosing 3 points from 32 points satisfying $x_1x_2x_3=x_4x_5x_6=x_1x_4x_7x_8=-1$ so that the columns for x_1, x_4 and x_7 are independent. For illustration, one such choice for $F_2=3$ FP's is

$$\begin{bmatrix} -1 & 1 & 1 & 1 & -1 & 1 & 1 & 1 \\ \\ 1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 \\ \\ 1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 \end{bmatrix}.$$

A general series of F_1 and F_2 FP's satisfying (5) is given below.

I. $k=3t-1$, $t\geq 2$ and t is an integer.

The F_u, $u=1,2$, FP's satisfying

$$x_1x_2x_3=\ldots=x_{3t-5}x_{3t-4}x_{3t-3}=-1,\ t\geq 2,$$

$$x_1x_4x_7x_8=\ldots=x_{t+3[t/3]-5}x_{t+3[t/3]-2}x_{3t-2}x_{3t-1}=(3-2u), t\geq 3,$$

II. $k = 3t,\ 3t+1, t \geq 1$ and t is an integer.

The $F_u, u = 1, 2$, FP's satisfying

$$x_1x_2x_3 = \ldots = x_{3t-2}x_{3t-1}x_{3t} = -1,\ t \geq 1,$$

$$x_1x_4x_7x_8 = \ldots = x_{t+3[t/3]-5}x_{t+3[t/3]-2}x_{3t-2}x_{3t-1} = (3-2u), t \geq 3, \tag{6}$$

where $[t/3]$ is the greatest integer in $(t/3)$.

There are some variations of the general series presented in (6). For $k = 10, 11$ and 12, the $F_u,\ u = 1, 2$, FP's satisfying

$$\begin{aligned} x_1x_2x_3 &= x_4x_5x_6 = x_7x_8x_9 = x_{10}x_{11}x_{12} = -1, \\ & x_1x_4x_7x_{10} = x_2x_5x_8x_{11} = (3-2u). \end{aligned} \tag{7}$$

The plans in (7) have smaller values of p_2 in comparison to the plans given in (6).

The other plans for $k = 4, 5$ and 6 are given below for F FP's under (3).

$$\begin{array}{lll} k = 4, & & x_4 = -1, F = 8. \\ k = 5, & \text{Plan I.} & x_1x_2x_3x_4x_5 = -1, \\ & \text{Plan II.} & x_5 = -1, \\ & & F = 16 \text{ for both plans.} \\ k = 6, & & x_1x_2x_3x_4x_5 = x_6 = -1, F = 16. \end{array} \tag{8}$$

The $(F_1 + F_2)$ FP's in (6) and (7) and the F FP's in (8) form OFP's under (3). It follows from (6) that the F_1 and F_2 FP's for $k = 3t+1$, are in fact twice of their counterparts for $k = 3t$. The $F_u,\ u = 1, 2$, FP's for $k = 3t+1$, can be obtained from the corresponding FP's for $k = 3t$ by adding 1 and -1, respectively in the last position.

3 Construction of Minimal Plans

The number of FP's discussed in Section 2 is in fact large. The idea of robustness of FP's against deletion of points [Ghosh (1979)] is now used in reducing the number of FP's. The idea turns out to be extremely powerful.

Definition. A set of F FP's is said to be robust against deletion of t (a positive integer) points if the parameters in $\underline{\beta}$ under (3) are still unbiasedly estimable with the remaining $(F-t)$ FP's.

For a robust set of F FP's, the rank of the resulting matrix remains p when t rows of X corresponding to t points are deleted. The following result of Ghosh (1979) is instrumental in determining the robustness of FP's.

Theorem 2. Let $Z((F-p)\times F)$ be a matrix with Rank $Z=(F-p)$ and $ZX=0$. A necessary and sufficient condition for a set of F FP's to be robust against deletion of t points is that Rank $Z^*=t$, where $Z^*((F-p)\times t)$ is the submatrix of Z corresponding to t points.

The following corollary is very useful in determining robustness by exploiting the design structure.

Corollary. Let $X'=[X_1', X_2']$ where $X_1(f_1\times p)$, $X_2(f_2\times p)$, $f_1+f_2=F$ and $X_1X_2'=0$. If the set of f_i FP's is robust against deletion of $t_i, i=1,2,$ points, then the set of F FP's is robust against deletion of (t_1+t_2) points and vice versa.

Proof. Note that Rank $X=$ Rank X_1+ Rank X_2 and the rest is clear.

Plans are now constructed for $4\le k\le 10$.

3.1 $(k = 4)$.

Two plans with 8 PF's in (6) and (8) are robust against deletion of any one point. The resulting plans with 7 FP's are denoted by Plan 4.1 and Plan 4.2. There are 8 possible plans for each of Plan 4.1 and Plan 4.2.

3.2 $(k = 5)$.

Consider $F_1 = 16$ FP's given in (6). Note that $p_2 = p_3 = 0$. The rows of the matrix $Z(5 \times 16)$ in Theorem 2 correspond to x_4, x_5, $x_1x_4x_5$, $x_2x_4x_5$, $x_3x_4x_5$ and the columns correspond to 16 points. It follows from the structure of Z that the 5 points with exactly one of the x_i's equal to 1 or -1, can be deleted. The resulting 11 points satisfy Rank $X = 11$ under (3).

Consider $F = 16$ FP's given in (8) satisfying $x_1x_2x_3x_4x_5 = -1$. The 11 points obtained by deleting the 5 points with exactly one of x_i's equal to -1, satisfy Rank $X = 11$.

Similarly consider $F = 16$ FP's given in (8) satisfying $x_5 = -1$. The 11 points obtained by deleting the 5 points with exactly one of x_i's equal to 1 or -1, also satisfy Rank $X = 11$.

Another set of 11 FP's is also given below.

$$\begin{bmatrix} -1 & -1 & -1 & -1 & -1 \\ 1 & -1 & -1 & -1 & 1 \\ 1 & -1 & 1 & 1 & -1 \\ 1 & 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 & -1 \\ 1 & -1 & 1 & -1 & 1 \\ 1 & 1 & -1 & -1 & 1 \\ 1 & 1 & 1 & -1 & -1 \\ 1 & 1 & -1 & -1 & -1 \\ 1 & -1 & 1 & -1 & -1 \\ -1 & 1 & 1 & -1 & -1 \end{bmatrix}. \qquad (9)$$

The above 4 plans with 11 FP's give identical X matrix under (3). The plan given in (9) is named as Plan 5.1.

3.3 $(k = 6)$.

The plans with 16 FP's given in (6) and (8), are called Plan 6.1 and Plan 6.2, respectively. These plans have the minimum number of points under (3).

Table 1
Efficiency Comparison for $k = 4$.

n_0	α	Plan	MCR	T	D	$V(r)$ $r=1$	$V(r)$ $r=\sqrt{2}$	$V(r)$ $r=\sqrt{k}$
0	1	4.1	4.878	10.194	1.668e-09	10.347	23.194	70.139
		4.2	1.473	7.194	1.657e-09	9.410	19.444	55.139
1	1	4.1	4.589	9.843	1.287e-09	10.345	24.131	73.262
		4.2	1.448	7.035	1.277e-09	9.409	20.387	58.284
	$\sqrt{k}$	4.1	1.259	3.709	1.716e-15	13.151	12.771	19.417
		4.2	1.255	3.523	1.810e-15	13.089	12.521	18.417
2	1	4.1	4.414	9.626	1.052e-09	10.535	25.237	76.818
		4.2	1.436	6.935	1.038e-09	9.582	21.427	61.576
	$\sqrt{k}$	4.1	0.988	3.084	8.602e-16	8.041	9.673	19.214
		4.2	0.783	2.898	9.081e-16	7.974	9.408	18.151

Table 2
Efficiency Comparison for $k = 5$.

n_0	α	Plan	MCR	T	D	$V(r)$ $r=1$	$V(r)$ $r=\sqrt{2}$	$V(r)$ $r=\sqrt{k}$
0	1	5.1	1.466	8.582	1.529e-15	12.569	27.215	107.236
		5.W	8.798	17.922	3.590e-15	15.687	39.664	185.937
		5.D.1	2.193	12.362	1.530e-15	13.812	32.263	135.896
		5.D.2	1.655	10.372	1.478e-15	13.153	29.611	119.819
	2	5.1	7.196	9.872	3.693e-22	86.340	62.111	40.007
		5.W	8.993	13.749	8.197e-22	99.125	73.250	64.625
		5.D.1	5.034	8.743	3.542e-22	63.743	49.056	45.410
		5.D.2	5.415	8.601	3.681e-22	67.528	50.861	41.694
1	1	5.1	1.461	8.521	1.273e-15	12.752	28.247	112.036
		5.W	8.686	17.757	3.010e-15	15.967	41.150	193.559
		5.D.1	2.193	12.334	1.340e-15	14.121	33.570	142.321
		5.D.2	1.655	10.339	1.277e-15	13.418	30.785	125.443
	$\sqrt{k}$	5.1	1.202	3.603	3.105e-24	18.413	17.264	28.441
		5.W	2.026	5.475	5.853e-24	19.064	19.878	44.821
		5.D.1	1.202	4.429	4.406e-24	18.775	18.530	35.667
		5.D.2	1.202	4.008	3.946e-24	18.613	17.922	32.022
	2	5.1	1.086	3.763	5.664e-23	15.833	15.672	31.425
		5.W	2.512	6.133	1.082e-22	16.840	19.014	51.936
		5.D.1	0.998	4.707	7.023e-23	15.704	16.996	40.283
		5.D.2	1.019	4.204	6.893e-23	15.629	16.297	35.790
2	1	5.1	1.458	8.477	1.104e-15	13.015	29.331	116.896
		5.W	8.606	17.636	2.503e-15	16.342	42.717	201.431
		5.D.1	2.193	12.314	1.204e-15	14.487	34.914	148.754
		5.D.2	1.655	10.312	1.116e-15	13.745	32.000	131.080
	$\sqrt{k}$	5.1	0.602	3.003	1.555e-24	10.780	12.098	28.273
		5.W	2.026	4.873	2.839e-24	11.461	14.831	45.398
		5.D.1	0.609	3.828	2.171e-24	14.487	34.914	148.754
		5.D.2	0.602	3.408	1.976e-24	10.989	12.785	32.017
	2	5.1	0.595	3.267	3.029e-23	10.228	12.197	31.965
		5.W	2.433	5.589	5.811e-23	11.109	15.568	53.119
		5.D.1	0.706	4.260	3.705e-23	10.558	13.822	41.278
		5.D.2	0.565	3.750	3.822e-23	10.379	13.038	36.564

Table 3

Efficiency Comparison for $k = 6$.

n_0	α	Plan	MCR	T	D	$V(r)$ $r=1$	$V(r)$ $r=\sqrt{2}$	$V(r)$ $r=\sqrt{k}$
0	1	6.1	1.032	9.587	4.440e-25	16.211	35.444	176.900
		6.2	1.032	9.227	4.397e-25	16.092	34.968	178.216
	$2^{5/4}$	6.1	49.889	51.966	1.021e-34	907.747	669.274	142.893
		6.2	78.443	80.402	1.060e-34	1414.564	1035.486	198.282
1	1	6.1	1.032	9.571	3.963e-25	16.501	36.505	183.189
		6.2	1.032	9.191	3.837e-25	16.323	35.990	184.269
	$\sqrt{k}$	6.1	1.168	3.181	9.466e-37	24.635	22.413	34.968
		6.2	1.168	3.069	6.196e-37	24.503	22.106	33.523
	$2^{5/4}$	6.1	1.149	3.226	2.352e-36	24.102	22.077	36.018
		6.2	1.164	3.123	1.574e-36	24.066	21.756	34.458
2	1	6.1	1.032	9.558	3.576e-25	16.83	37.593	189.482
		6.2	0.493	2.917	1.591e-34	11.473	13.781	43.875
	$\sqrt{k}$	6.1	0.584	2.597	4.733e-37	13.984	14.686	34.674
		6.2	0.585	2.486	3.103e-37	13.848	14.368	33.179
	$2^{5/4}$	6.1	0.582	2.659	1.191e-36	13.905	14.758	35.912
		6.2	0.587	2.546	7.938e-37	13.768	14.393	34.326

Table 4
Efficiency Comparison for $k = 7$.

n_0	α	Plan	MCR	T	D	$V(r)$ $r=1$	$V(r)$ $r=\sqrt{2}$	$V(r)$ $r=\sqrt{k}$
0	1	7.1	5.512	16.298	4.907e-33	22.472	53.007	387.556
		7.2	9.246	20.184	4.692e-33	23.886	58.608	448.182
		7.W	12.946	29.464	3.381e-31	27.260	72.179	625.412
		7.DL.1	58.800	77.428	2.741e-31	44.701	141.865	1467.853
		7.DL.2	24.485	40.931	2.022e-31	31.430	88.799	820.326
	$2^{6/4}$	7.1	13.232	17.672	1.470e-48	337.852	273.909	137.261
		7.2	8.486	13.296	1.395e-48	220.443	180.273	120.216
		7.W	27.740	33.611	1.041e-46	691.102	558.909	238.511
		7.DL.1	15.293	26.865	8.256e-47	199.638	180.609	375.441
		7.DL.2	10.871	21.521	6.534e-47	264.983	223.540	244.855
1	1	7.1	5.461	16.230	4.265e-33	22.785	54.212	397.247
		7.2	9.246	20.173	4.269e-33	24.296	60.047	460.609
		7.W	12.814	29.307	3.025e-31	27.668	73.798	639.998
		7.DL.1	58.800	77.418	2.505e-31	45.689	145.615	1508.606
		7.DL.2	24.433	40.864	1.842e-31	32.018	90.994	842.064
	$\sqrt{k}$	7.1	2.211	5.790	1.111e-48	32.817	32.804	95.299
		7.2	2.351	6.224	1.787e-48	33.064	33.591	103.159
		7.W	1.926	7.469	4.443e-47	33.491	35.403	126.286
		7.DL.1	16.170	21.684	1.127e-46	38.840	56.695	386.177
		7.DL.2	7.628	12.736	6.905e-47	35.467	43.276	222.437
	$2^{6/4}$	7.1	2.208	5.540	1.139e-49	30.782	31.390	92.410
		7.2	2.211	5.810	1.640e-49	29.711	30.850	97.652
		7.W	1.879	7.081	4.193e-48	32.691	34.876	120.560
		7.DL.1	15.286	20.390	1.084e-47	34.704	52.245	364.600
		7.DL.2	7.363	12.121	6.581e-48	32.605	40.718	213.040
2	1	7.1	5.419	16.178	3.969e-33	23.141	55.454	407.083
		7.2	9.246	20.164	3.907e-33	24.735	61.508	473.040
		7.W	12.708	29.182	2.751e-31	28.125	75.472	654.981
		7.DL.1	58.800	77.409	2.292e-31	46.706	149.388	1549.362
		7.DL.2	24.391	40.807	1.660e-31	32.640	93.221	863.930
	$\sqrt{k}$	7.1	2.211	5.217	5.466e-49	18.679	22.148	96.340
		7.2	2.351	5.652	8.937e-49	18.934	22.956	104.411
		7.W	1.926	6.897	2.221e-47	19.371	24.817	128.164
		7.DL.1	16.170	21.113	5.644e-43	24.866	46.684	395.079
		7.DL.2	7.628	12.165	3.456e-47	21.401	32.903	226.913
	$2^{6/4}$	7.1	2.186	5.035	5.941e-50	18.072	21.535	92.825
		7.2	2.211	5.343	8.731e-50	17.878	21.771	98.628
		7.W	1.804	6.528	2.139e-48	19.076	24.274	121.153
		7.DL.1	15.286	19.935	5.806e-48	23.331	44.025	372.918
		7.DL.2	7.351	11.623	3.467e-48	20.438	31.530	216.696

Table 5
Efficiency Comparison for $k = 8$.

n_0	α	Plan	MCR	T	D	$V(r)$ $r=1$	$V(r)$ $r=\sqrt{2}$	$V(r)$ $r=\sqrt{k}$
0	1	8.1	4.507	20.696	1.511e-44	28.511	68.598	640.190
		8.2	4.454	21.007	1.436e-44	28.633	68.993	635.767
		8.3	4.456	21.043	1.707e-44	28.645	69.040	636.517
		8.4	4.455	21.038	1.419e-44	28.645	69.040	636.517
		8.DL.1	9.047	29.812	4.568e-43	31.934	82.196	846.954
1	1	8.1	4.505	20.648	1.381e-44	28.869	69.885	653.340
		8.2	4.454	21.000	1.336e-44	29.038	70.349	649.878
		8.3	4.456	21.035	1.568e-44	29.050	70.396	650.642
		8.4	4.455	21.032	1.326e-44	29.050	70.397	650.644
		8.DL.1	9.047	29.802	4.223e-43	32.411	83.841	865.722
	$\sqrt{k}$	8.1	1.212	6.047	7.592e-64	41.226	40.897	132.250
		8.2	1.141	6.286	1.191e-63	41.400	41.400	138.000
		8.3	1.126	6.260	1.074e-63	41.391	41.364	137.425
		8.4	1.136	6.291	1.111e-63	41.403	41.412	138.192
		8.DL.1	1.956	8.756	4.616e-62	42.338	45.166	198.415
2	1	8.1	4.504	20.608	1.272e-44	29.259	71.200	666.615
		8.2	4.454	20.994	1.242e-44	29.467	71.722	663.992
		8.3	4.456	21.029	1.462e-44	29.479	71.771	664.769
		8.4	4.455	21.025	1.227e-44	29.479	71.771	664.772
		8.DL.1	9.046	29.793	3.885e-43	32.912	85.505	884.496
	$\sqrt{k}$	8.1	1.212	5.485	3.802e-64	23.078	26.707	133.558
		8.2	1.141	5.724	5.962e-64	23.255	27.221	139.433
		8.3	1.089	5.698	5.379e-64	23.246	27.184	138.846
		8.4	1.135	5.728	5.500e-64	23.258	27.233	139.629
		8.DL.1	1.956	8.194	2.310e-62	24.214	31.069	201.162

Table 6
Efficiency Comparison for $k = 9$.

n_0	α	Plan	MCR	T	D	$V(r)$ $r=1$	$V(r)$ $r=\sqrt{2}$	$V(r)$ $r=\sqrt{k}$
0	1	9.1	1.873	17.967	2.486e-58	32.383	74.351	746.289
		9.2	2.755	18.965	2.140e-58	32.767	75.889	777.443
		9.3	3.946	20.151	2.182e-58	33.224	77.716	814.438
		9.4	4.329	20.276	2.023e-58	33.272	77.908	818.332
		9.DL.1	60.213	97.206	6.975e-56	62.861	196.261	3214.621
	$2^{7/4}$	9.1	2.895	8.239	1.689e-83	113.013	103.383	197.908
		9.2	2.866	9.236	1.461e-83	113.398	104.921	229.062
		9.3	3.962	10.424	1.382e-83	113.854	106.748	266.057
		9.4	4.986	10.552	1.572e-83	113.902	106.941	269.952
		9.DL.1	13.807	27.932	5.415e-81	106.562	122.140	819.291
1	1	9.1	1.873	17.960	2.287e-58	32.757	75.532	759.839
		9.2	2.755	18.959	1.983e-58	33.148	77.099	791.565
		9.3	3.946	20.145	2.017e-58	33.613	78.959	829.232
		9.4	4.328	20.269	1.871e-58	33.662	79.153	833.166
		9.DL.1	60.213	97.200	6.533e-56	63.789	199.657	3273.031
	$\sqrt{k}$	9.1	1.613	6.873	1.416e-81	50.935	51.013	195.564
		9.2	2.714	7.971	1.293e-81	51.366	52.735	230.433
		9.3	3.927	9.155	1.617e-81	51.827	54.582	267.838
		9.4	3.636	8.706	1.345e-81	51.653	53.884	253.698
		9.DL.1	14.948	27.989	6.016e-79	59.203	84.086	865.279
	$2^{7/4}$	9.1	1.662	6.327	5.082e-84	37.336	40.060	87.841
		9.2	2.717	7.409	4.386e-84	37.761	41.758	222.226
		9.3	3.929	8.588	4.149e-84	38.223	43.608	259.672
		9.4	3.777	8.234	4.732e-84	38.083	43.047	248.312
		9.DL.1	13.805	25.998	1.784e-81	43.582	69.630	646.342
2	1	9.1	1.873	17.957	2.191e-58	33.150	76.730	773.391
		9.2	2.755	18.954	1.850e-58	33.549	78.324	805.688
		9.3	3.946	20.141	1.906e-58	34.022	80.218	844.028
		9.4	4.327	20.264	1.769e-58	34.071	80.414	848.004
		9.DL.1	60.213	97.193	6.096e-56	64.737	203.070	3331.446
	$\sqrt{k}$	9.1	1.613	6.318	7.088e-82	28.283	32.813	197.462
		9.2	2.714	7.416	6.475e-82	28.721	34.566	232.953
		9.3	3.927	8.599	8.089e-82	29.191	36.446	271.027
		9.4	3.636	8.151	6.732e-82	29.013	35.736	256.634
		9.DL.1	14.948	27.435	3.015e-79	36.699	66.476	879.136
	$2^{7/4}$	9.1	1.620	5.990	2.987e-84	24.049	29.071	188.742
		9.2	2.714	7.086	2.578e-84	24.487	30.823	224.219
		9.3	3.927	8.264	2.429e-84	24.957	32.703	262.294
		9.4	3.653	7.826	2.792e-84	24.780	31.996	247.981
		9.DL.1	13.805	25.617	1.069e-81	31.334	59.890	822.696

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Book Edited:

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ADVANCES IN RELIABILITY, North-Holland, Amsterdam, 1993.

Book Chapters:

1. Klein, John P. and Goel, Prem K., Editors, Survival Analysis: State of the Art, Kluwer Academic Publishers, Dordrecht, 1992.

   •Basu, A. P., Life Testing and Reliability Estimation with Asymmetric Loss Function.

2. Sen, Pranab K. and Salama, Ibrahim A., Editors, Order Statistics and Nonparametrics: Theory and Applications.

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